Discrete Random Variables

4.3 Standard Deviation of a Discrete Random Variable

1. A pair of six-sided die is rolled four times. We record the number of sevens rolled over the four rolls. Calculate the standard deviation for the number of sevens rolled. Would it be unusual for someone to roll 3 or 4 sevens out of four rolls? (note: the mean for the probability distribution is $E(X) = 0.667$)

X	P(X)
0	0.482
1	0.386
2	0.116
3	0.015
4	0.001

2. Converting the above probability distribution into a game, players can risk \$4 to bet that a seven will not be rolled in four throws of the dice. If the player wins, he will be paid \$7 (a three-dollar profit), but if the player loses he/she loses his/her \$4 bet. What is the standard deviation for the outcome of this game?

3. The table below is the probability distribution for the number of televisions in American homes. Find the standard deviation for the number of TVs in American homes. What is the z-score for a household with no television? Is it unusual to not have a TV in America? (note: the mean is 2.75 TVs)

X	P(X)
0	0.02
1	0.15
2	0.29
3	0.26
4	0.16
5	0.12

Answers:

1. The standard deviation is: $\sigma = 0.746$. Based on the following interval it would be unusual to roll more than two sevens out of four rolls: $(\mu - 2\sigma, \mu + 2\sigma) = (0.667 - 2*0.746, 0.667 + 2*0.746) = (-0.825, 2.159)$.

X	P(X)	X*P(X)	$X^2 * P(X)$
0	0.482	0	0
1	0.386	0.386	0.386
2	0.116	0.232	0.464
3	0.015	0.045	0.135
4	0.001	0.004	0.016
		0.667	1.001

2. The standard deviation is $\sigma = 3.50$, but this is not very useful since there are only two outcomes for a trial. We know either a person will win 3 dollars or will lose 4 dollars. The standard deviation is not needed to capture the variation. It is easier to see the variation by simply viewing the win and loss probabilities.

3. The standard deviation is: $\sigma = 1.284$. The z-score is $z = \frac{(0 - 2.75)}{1.284} = -2.14$. It would be unusual to not have a TV in America since the z-score is -2.14.

X	P(X)	X*P(X)	$X^2 * P(X)$
0	0.02	0	0
1	0.15	0.15	0.15
2	0.29	0.58	1.16
3	0.26	0.78	2.34
4	0.16	0.64	2.56
5	0.12	0.60	3.00
		2.75	9.21